Journal of
Nanophotonics

SPIEDigitalLibrary.org/jnp

Designing super-resolution metalenses by the combination of metamaterials and nanoscale plasmonic waveguide couplers

Changbao Ma
Zhaowei Liu

Designing super-resolution metalenses by the combination of metamaterials and nanoscale plasmonic waveguide couplers

Changbao Ma and Zhaowei Liu
University of California, San Diego, Department of Electrical and Computer Engineering,
La Jolla, California 92093-0407
zhaowei@ece.ucsd.edu

Abstract. We recently demonstrated a phase compensated metalens that cannot only achieve super-resolution, but also possesses the Fourier transform capability. The metalens consists of a metamaterial slab and a plasmonic waveguide coupler (PWC). We have now ascertained the requirements for the metamaterial and the detailed design principles for the PWCs. Simulations of metalenses with a new type of PWC geometry have confirmed that the new metalenses also possess super-resolution and the Fourier transform function. The hyperbolic metalens shows an anomalous focus shifting behavior, which may be used to design exotic optical systems with new functionalities.
[DOI: 10.1117/1.3579159]

Keywords: super-resolution; metalens; metamaterial; plasmonic waveguide.

Paper 10090SSR received Nov. 30, 2010; accepted for publication Mar. 17, 2011; published online Apr. 26, 2011.

1 Introduction

Lenses are the most fundamental and widely used optical elements in optical imaging systems. They make use of curved interfaces to refract and focus light beams. The resolution of a lens has long been believed to be limited to half the working wavelength $\sim\lambda/2$ due to the diffraction of light waves. Better resolution has been pursued and mainly remained in the immersion technologies for centuries since the first invention of water immersion lens by Giovanni Battista Amici in 1850.[1] The immersion techniques can improve the resolution to $\sim\lambda/(2n)$, where n is the refractive index of the immersion liquid or the lens material in solid immersion.[2] The major limitation of this technique is the unavailability of high refractive index materials in nature. In order to achieve even better resolution, optical materials that can cover a higher wavevector band are essential. As have been proposed and demonstrated using artificial metamaterials,[3–5] a number of superlensing approaches emerged in the past decade to achieve resolution beyond the diffraction limit.[6–15] However, none of them can focus a plane wave to a spot because of the lack of phase compensation consideration, making them work distinctly from a conventional lens. This plane wave focusing, also called Fourier transform, is one of the fundamental functions of a lens and the basis of Fourier optics, playing vital roles in optical system design and information/image processing. We recently proposed the phase compensated superlens concept and numerically demonstrated it using the metalenses based on plasmonic metamaterials.[16] They can achieve super-resolution like other superlenses, but for the first time possess the Fourier transform capability.

The proof of concept of the metalens was previously demonstrated using the combination of a metamaterial and a plasmonic waveguide coupler (PWC).[16–19] In this work, we show the detail of designing a metalens, including the general requirements for the metamaterial of a metalens, the phase calculations originating from the propagation in the waveguides and the metamaterial.

1934-2608/2011/$25.00 © 2011 SPIE

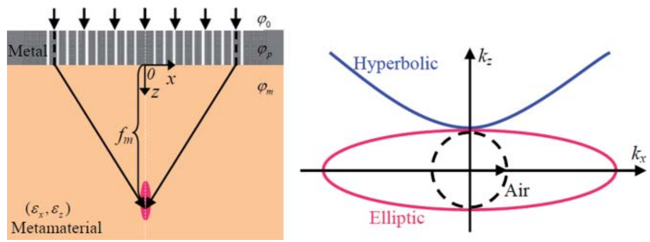

Fig. 1 (a) Schematic of a metalens. (b) Non-flat EFCs of the metamaterial for a metalens, exampled by an ellipse and a hyperbola.

In contrast to tuning the width of the same high waveguides to achieve phase compensation for plane wave focusing as in the previously demonstrated metalenses,[16] we design new elliptic and hyperbolic metalenses by tuning the height of plasmonic waveguides with the same width.

2 General Requirements for Metamaterial and Phase Calculation Methods

2.1 *General Requirements for the Metamaterial of a Metalens*

A metalens consists of a metamaterial slab and a phase compensation element, which is particularly a PWC, i.e., a nonperiodic plasmonic waveguide array, in this case, as schematically shown in Fig. 1(a). The metamaterial is required to have high transverse wavevector coverage to achieve the super resolving power. Although both the permittivity and permeability of metamaterials can be designed for novel devices and phenomena,[3,4,20] it is prohibitively difficult to realize the tuning of the permeabilities of bulky metamaterials at visible light frequencies. In this work we focus on plasmonic metamaterials, with tuning only the permittivities because of their relatively low loss and availability of fabrication techniques for bulky materials.[5,13] The equi-frequency contour (EFC) of the metamaterial of a metalens needs to be non-flat, as is exampled by an ellipse and a hyperbola in Fig. 1(b) (illustration in 2D for simplicity), which is in contrast to the highly flat or ideally straight linear EFC of a hyperlens.[10,12,15,21]

The non-flat EFC of the metamaterial is essential for a metalens, because waves with various wavevectors in such a metamaterial can propagate in different directions such that the rays of an incident plane wave at different positions can propagate toward a point (the focus) in different directions. A perfect focus is obtained if all these rays coming in different directions have the same phase thus constructively interfere at the focal point.

As is illustrated in our original metalens demonstration, for such a metamaterial with high wavevector coverage, a PWC is unique to the high wavevector coupling and phase compensation.[16] In the following, we formulate the equations for calculating the phase delay caused by the light propagation in the waveguides (φ_p) and the anisotropic metamaterial (φ_m), so that the phase compensation and thus the PWC can be accurately computed.

2.2 *Phase Retardation by Plasmonic Waveguides*

The PWC in a metalens is a nonperiodic metal-insulator-metal (MIM) waveguide[17–19,22] array that is designed to provide phase compensation to the rays in different directions for focusing inside a metamaterial at the nanoscales, because the modal index β of an MIM waveguide can be tuned in a large range by material and geometric parameters as a result of the plasmonic effect. Figure 2 shows the schematic of a MIM waveguide above a metamaterial and a large β range simply tuned by varying the width w of its insulator layer.

Considering normal plane wave incidence, light is coupled into the eigenmode of the MIM waveguide, then propagates along the waveguide and undergoes Fabry–Perot-like multiple

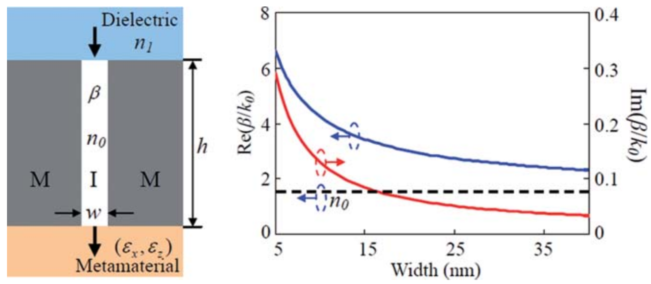

Fig. 2 (a) Schematic of a MIM waveguide above a metamaterial. (b) Dependence of the modal index of a nanoscale MIM waveguide on the width *w* of the insulator layer. The upper curve represents the real part and the lower curve represents the imaginary part. Data obtained using infinitely thick silver and a dielectric with a refractive index $n_0 = 1.5$. The dashed line represents the propagation constant of a plane wave in the core dielectric material n_0.

round-trips due to the reflections at the inlet and outlet of the waveguide. The phase change of the light finally transmitted to the metamaterial at the outlet (the interface between the waveguide and the metamaterial) can be derived as

$$\varphi_p = \varphi_1 + \varphi_2 + \Omega + \varphi_w, \tag{1}$$

where $\varphi_1 = \arg[2n_1/(\beta + n_1)]$ and $\varphi_2 = \arg[2\beta/(\beta + \sqrt{\varepsilon_x})]$ are the phase change at the inlet and the outlet of the waveguide respectively, $\Omega = \arg\{1-[(\beta - n_1)/(\beta + n_1)][(\beta - \sqrt{\varepsilon_x})/(\beta + \sqrt{\varepsilon_x})]e^{-i2\beta h}\}^{-1}$ is the phase change resulting from the Fabry–Perot-like oscillations, and $\varphi_w = \mathrm{Re}(\beta h)$ is the phase retardation of the waveguide eigenmode propagation. In the expression of Ω above, n_1 is the refractive index of the dielectric above the waveguide and ε_x is the permittivity of the metamaterial in the x direction. Numerical calculations show that φ_w is usually much larger than φ_1, φ_2, and Ω in practical design, thus φ_p can be approximated as $\mathrm{Re}(\beta h)$.

2.3 *Phase Retardation in the Metamaterial*

The phase delay in a uniaxial anisotropic metamaterial φ_m may be derived by analyzing the relation of the phase and group velocities of a ray between two points in the material. Figure 3 shows a diagram for such a case, with one of the points located at the origin. Because of the material anisotropy, group and phase velocities are in different directions, for which the relation is $\tan\theta = (\varepsilon_z/\varepsilon_x)\tan\phi$, where θ and ϕ are the angle of the phase velocity (the wavevector $\vec{k}$ direction) and the group velocity (the ray direction $\vec{s}$) with respect to the z axis,

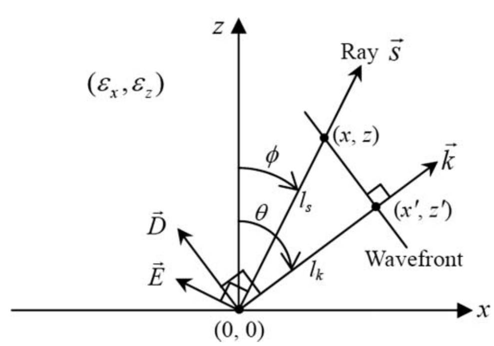

Fig. 3 Group velocity (ray direction) and phase velocity (wavevector $\vec{k}$ direction) of a ray in a uniaxial anisotropic medium with permittivities $(\varepsilon_x, \varepsilon_z)$.

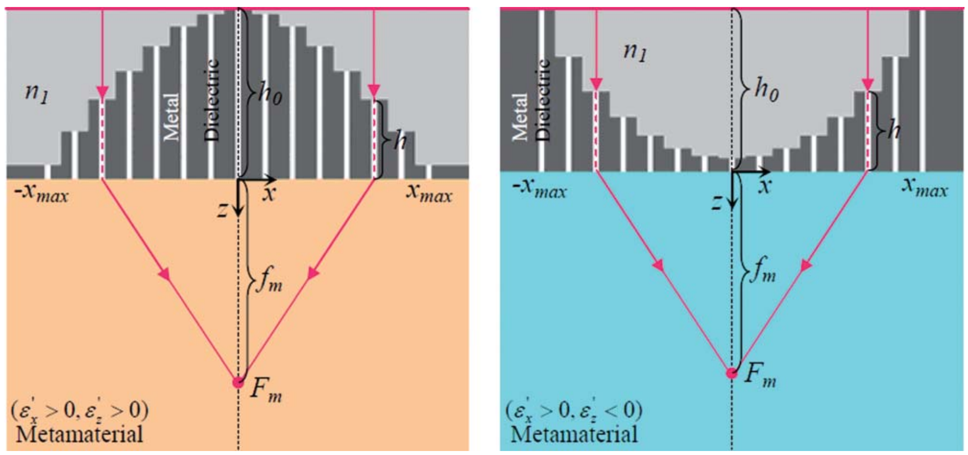

Fig. 4 Schematics of (a) an elliptic metalens and (b) a hyperbolic metalens with a height tuning PWC.

respectively. When the ray propagates from the origin (0, 0) to point (x, z), the phase velocity propagates to point (x', z'); both points are on the same wavefront that is normal to the $\vec{k}$ direction, as shown in Fig. 3. Because the effective index in the $\vec{k}$ direction can be written as $n_{eff}(\theta) = 1/\sqrt{\cos^2\theta/\varepsilon_x + \sin^2\theta/\varepsilon_z}$, the phase difference Δp between points (0, 0) and (x', z') is expressed as $\Delta p = n_{eff}(\theta)k_0 l_k$, where $l_k = l_s\cos(\theta\ \text{-}\phi)$ is the distance between (0, 0) and (x', z'), k_0 is the wavevector in vacuum, and $l_s = \sqrt{x^2 + z^2}$ is the distance between (0, 0) and (x, z). After mathematic manipulations, we have $\Delta p = k_0\sqrt{\varepsilon_x z^2 + \varepsilon_z x^2}$. So the phase delay φ_m can be written as

$$\varphi_m = k_0\sqrt{\varepsilon_x f_m^2 + \varepsilon_z x^2} \tag{2}$$

with f_m being the focal length of the metalens in the metamaterial. Equation (2) is the general form of φ_m that can be used for both elliptically and hyperbolically dispersive metamaterials. It can be approximated by $k_0\sqrt{\varepsilon_x' f_m^2 + \varepsilon_z' x^2}$, with the prime taking the real part of the complex permittivities, when the metamaterial is not working under resonant conditions.

As shown above, we have given the requirements of the metamaterial for the metalens purpose, and derived the expressions for calculating the phase changes originating from the light propagation in the MIM waveguides φ_p and the metamaterial φ_m. Using the metalens phase condition for plane wave focusing, i.e., $\varphi_p + \varphi_m = \varphi_{\text{const}} + 2l\pi$ with φ_{const} being a constant phase and l being an integer, the MIM waveguide properties at different locations required for a metalens can be calculated and thus a metalens with a focal length f_m can be designed.[16]

Although both the material and geometric parameters of the PWCs can be tuned to satisfy the phase condition in designing a metalens, varying only the geometric parameters may result in easier fabrication. One approach is to tune the width of the waveguides to attain different propagation constants that satisfy the phase condition, while maintaining the same waveguide height everywhere. This method was employed in our original metalens demonstration.[16] A metalens can also be designed by using MIM waveguides with same width (thus same propagation constant) but different height h at different locations. In the following, we design new metalenses with height tuning PWCs.

3 Metalens with Height Tuning PWC

Figure 4 shows the schematics of an elliptic metalens and a hyperbolic metalens with a height tuning PWC. We assume designing an elliptic metalens and a hyperbolic metalens with a focal length f_m in the metamaterial and $x \in [-x_{\max}, x_{\max}]$. After a metamaterial with $(\varepsilon_x, \varepsilon_z)$ is chosen according to the material requirements for a metalens in Sec. 2.1, the remaining task is to determine the height profile of the PWC, of which the propagation constant of the MIM waveguides is β. When a plane wave is incident on the metalenses from the top, the waveguide eigenmode is launched and the light continues to propagate to the interface between the waveguides and metamaterial. Because each waveguide is at the deep subwavelength scale, the output can be considered as a line source, which contains a broad range of wavevectors. Therefore, high frequency spatial information can be coupled into the metamaterial and contribute to the focus at F_m. Otherwise, wavevectors higher than a certain value cannot be coupled into and out of the metamaterial due to the light cone issue caused by total internal reflection.[16] The phase condition for constructive interference at F_m results in the following equation

$$\varphi_{tot} = k_0\sqrt{\varepsilon_x' f_m^2 + \varepsilon_z' x^2} + k_0\beta h + k_0(h_0 - h)n_1, \tag{3}$$

where the total phase delay $\varphi_{tot} = k_0\sqrt{\varepsilon_x' f_m^2} + k_0\beta h_0$ for the elliptic metalens and $\varphi_{tot} = k_0\sqrt{\varepsilon_x' f_m^2 + \varepsilon_z' x_{\max}^2} + k_0\beta h_0$ for the hyperbolic one. Rewriting Eq. (3), with setting $n_1 = 1$ for air, results in the height profile of the waveguides at different x positions,

$$h(x) = \left(\varphi_{tot} - k_0\sqrt{\varepsilon_x' f_m^2 + \varepsilon_z' x^2} - k_0 h_0\right)/[k_0(\beta - 1)] \tag{4}$$

Equation (4) can be used to determine the height profile for both the elliptic and the hyperbolic metalenses. Note that the negative ε_z' imposes a limit for the width of a hyperbolic metalens, i.e., $x_{\max} < f_m\sqrt{-\varepsilon_x'/\varepsilon_z'}$.

Simulations have been carried out to verify the analysis above for both cases at the wavelength of 633 nm. Figure 5(a) shows the simulated power profile of an elliptic metalens with a PWC having different waveguide heights determined using Eq. (4), illuminated by a normal plane wave. The permittivity of the elliptic metamaterial is $\varepsilon_x = 5.1 + 0.1$i and $\varepsilon_z = 16 + 0.08$i. The MIM waveguide structure is Ag/Alumina/Ag with a constant core width of 20 nm and a constant spacing of 60 nm (i.e., the period is 80 nm). The PWC consists of 43 MIM waveguides, of which the height varies gradually from 50 nm at the edges to 1332 nm at the center (h_0). The focal length of the elliptic metalens is $f_m = 2.2$ μm. It achieves a focus with a width of 74 nm ($\sim\lambda/8.6$), measured by the full width at half maximum, for the normal plane wave incidence. When the incident light is tilted, the focus is shifted accordingly, as shown in Fig. 5(b). Because of the elliptic dispersion, the shifting direction is similar to that in a conventional lens case and the previously illustrated elliptic metalens with a width tuning PWC. The simulation in Fig. 5(b) shows a shift of 117 nm for an incident plane wave tilted at the angle of 16.4 deg with respect to the z axis.

The elliptic metalens with a height tuning PWC shows its super-resolution performance with a focus shifting behavior similar to that of a conventional lens and the elliptic metalens with a width tuning PWC. The hyperbolic metalens with a height tuning PWC is expected to behave similar to the previously illustrated hyperbolic metalens with a width tuning PWC; thus the focus shifting behavior of such a hyperbolic metalens will be opposite to that of a conventional lens. The simulations for a hyperbolic metalens designed using Eq. (4) shown in Figs. 5(c) and 5(d) confirm this conclusion, with Figs. 5(c) and 5(d) being illuminated with a normal and a tilted plane wave, respectively, in which the opposite focus shifting behavior is clearly seen. The permittivity of the hyperbolic metamaterial is $\varepsilon_x = 8.1 + 0.1$i and $\varepsilon_z = -12.5 + 0.3$i. The PWC of the hyperbolic metalens is composed of 33 Ag/Alumina/Ag waveguides with the same width parameters as those in the elliptic metalens above. However, the height profile of the waveguides of the hyperbolic metalens, varying from 50 nm at the center to 851 nm at the edges (h_0), is concave, which is opposite to the convex one of the elliptic metalens. The focal length of the hyperbolic metalens is $f_m = 1.9$ μm. It achieves a focus with a width of 90 nm ($\sim\lambda/7.0$)

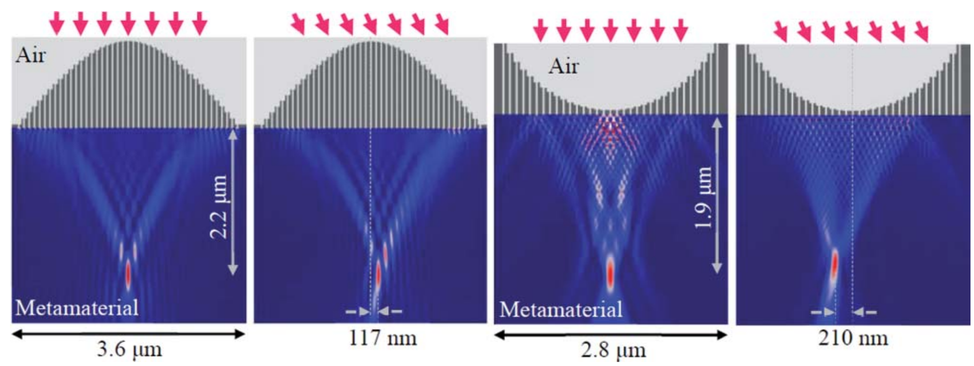

Fig. 5 Simulations of an elliptic metalens with a height tuning PWC, illuminated by (a) a normal and (b) a tilted plane wave, respectively, and simulations of a hyperbolic metalens with a height tuning PWC, illuminated by (c) a normal and (d) a tilted plane wave, respectively.

for the normal plane wave incidence. The simulation in Fig. 5(d) shows a shift of 210 nm for an incident plane wave tilted at the angle of 39.4 deg with respect to the vertical direction.

4 Conclusions and Discussions

We have shown that a metalens requires a metamaterial that supports high wavevector coverage. Such a metamaterial can be conveniently designed using, for example, multilayer of alternating metallic/dielectric composites and metallic nanowires in a dielectric matrix, of which the material permittivities can be easily estimated using the effective medium theory.[23,24] After the metamaterial for a metalens is chosen, a PWC and thus the metalens can be readily designed using Eq. (4).

As is demonstrated above, the elliptic and hyperbolic metalenses with height tuning PWCs can achieve super-resolution and focus plane waves. While the elliptic metalens achieves focusing by a convex profile PWC and shows a normal focus shifting behavior, the hyperbolic one achieves focusing by a concave profile PWC and shows an exotic focus shifting behavior. Because the metalenses can focus plane waves, they have the Fourier transform capability. Therefore, the newly designed metalenses with height tuning PWCs possess all the properties of the metalenses, adding a new member to the metalens family.

Although the presented height tuning PWC-based metalenses, as compared to the previously demonstrated width tuning PWC-based metalenses, are not planar, the width of the waveguides is equal and may avoid using narrow MIM waveguides. It should also be noted that the height tuning PWCs have similarities with the shapes of the metamaterial immersion lenses (MILs)[25] in that both the elliptic metalens and the elliptic MIL need a convex PWC and interface, respectively, and both the hyperbolic metalens and the hyperbolic MIL need a concave PWC and interface, respectively. The major difference lies in that the PWC offers more flexibility to tune its height using different waveguide material combinations, thus the PWC may be shorter than the curved interfaces of the MILs. It is worth commenting that zone plates may also be used as the couplers for the metalenses. However, because the openings of a zone plate have to be located at discrete zones, the density or the number of zones is low, thus lowering the overall efficiency of the metalenses. We note that metallic wire arrays have also been proposed for superlensing and transmission of images with subwavelength resolution.[26,27] We finally remark that although the height tuning PWC-based metalenses are illustrated at a visible light frequency in 2D, they can be extended to other electromagnetic waves, and even acoustic waves in both 2D and 3D.

Acknowledgments

This work was supported by funding from the University of California at San Diego research start-up funds and the research grant from the NSF-ECCS under Grant No. 0969405.

References

1. M. Davidson, "Pioneers in optics: Giovanni Battista Amici and Girolamo Cardano," *Microsc. Today* **18**, 50–52 (2010).
2. G. Kino, "The solid immersion lens," *Proc. SPIE* **3740**, 2–6 (1999).
3. D. R. Smith, J. B. Pendry, and M. C. K. Wiltshire, "Metamaterials and negative refractive index," *Science* **305**, 788–792 (2004).
4. V. Shalaev, "Optical negative-index metamaterials," *Nat. Photon.* **1**, 41–48 (2007).
5. J. Yao, Z. Liu, Y. Liu, Y. Wang, C. Sun, G. Bartal, A. M. Stacy, and X. Zhang, "Optical negative refraction in bulk metamaterials of nanowires," *Science* **321**, 930 (2008).
6. J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett.* **85**, 3966–3969 (2000).
7. N. Fang, H. Lee, C. Sun, and X. Zhang, "Sub-diffraction-limited optical imaging with a silver superlens," *Science* **308**, 534–537 (2005).
8. A. Salandrino and N. Engheta, "Far-field subdiffraction optical microscopy using metamaterial crystals: Theory and simulations," *Phys. Rev. B* **74**, 075103 (2006).
9. T. Taubner, D. Korobkin, Y. Urzhumov, G. Shvets, and R. Hillenbrand, "Near-field microscopy through a SiC superlens," *Science* **313**, 1595 (2006).
10. Z. Jacob, L. Alekseyev, and E. Narimanov, "Optical hyperlens: Far-field imaging beyond the diffraction limit," *Opt. Express* **14**, 8247–8256 (2006).
11. I. Smolyaninov, Y. Hung, and C. Davis, "Magnifying superlens in the visible frequency range," *Science* **315**, 1699–1701 (2007).
12. Z. Liu, H. Lee, Y. Xiong, C. Sun, and X. Zhang, "Far-field optical hyperlens magnifying sub-diffraction-limited objects," *Science* **315**, 1686 (2007).
13. Z. Liu, S. Durant, H. Lee, Y. Pikus, N. Fang, Y. Xiong, C. Sun, and X. Zhang, "Far-field optical superlens," *Nano Lett.* **7**, 403–408 (2007).
14. X. Zhang and Z. Liu, "Superlenses to overcome the diffraction limit," *Nat. Mater.* **7**, 435–441 (2008).
15. C. Ma, R. Aguinaldo, and Z. Liu, "Advances in the hyperlens," *Chin. Sci. Bull.* **55**, 2618–2624 (2010).
16. C. Ma and Z. Liu, "A super-resolution metalens with phase compensation mechanism," *Appl. Phys. Lett.* **96**, 183103 (2010).
17. H. Shi, C. Wang, C. Du, X. Luo, X. Dong, and H. Gao, "Beam manipulating by metallic nano-slits with variant widths," *Opt. Express* **13**, 6815–6820 (2005).
18. L. Verslegers, P. B. Catrysse, Z. Yu, J. S. White, E. S. Barnard, M. L. Brongersma, and S. Fan, "Planar lenses based on nanoscale slit arrays in a metallic film," *Nano Lett.* **9**, 235–238 (2008).
19. Z. Sun and H. K. Kim, "Refractive transmission of light and beam shaping with metallic nano-optic lenses," *Appl. Phys. Lett.* **85**, 642–644 (2004).
20. W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Optical cloaking with metamaterials," *Nat. Photon.* **1**, 224–227 (2007).
21. H. Lee, Z. Liu, Y. Xiong, C. Sun, and X. Zhang, "Development of optical hyperlens for imaging below the diffraction limit," *Opt. Express* **15**, 15886–15891 (2007).
22. J. Dionne, L. Sweatlock, H. Atwater, and A. Polman, "Plasmon slot waveguides: Towards chip-scale propagation with subwavelength-scale localization," *Phys. Rev. B* **73**, 035407 (2006).
23. S. A. Ramakrishna, J. B. Pendry, M. C. K. Wiltshire, and W. J. Stewart, "Imaging the near field," *J. Mod. Opt.* **50**, 1419–1430 (2003).

24. J. Pendry, A. Holden, D. Robbins, and W. Stewart, "Low frequency plasmons in thin-wire structures," *J. Phys.: Condens. Matter* **10**, 4785–4810 (1998).
25. C. Ma and Z. Liu, "Focusing light into deep subwavelength using metamaterial immersion lenses," *Opt. Express* **18**, 4838–4844 (2010).
26. P. A. Belov, C. R. Simovski, and P. Ikonen, "Canalization of subwavelength images by electromagnetic crystals," *Phys. Rev. B* **71**, 193105 (2005).
27. P. A. Belov, Y. Zhao, S. Tse, P. Ikonen, M. G. Silveirinha, C. R. Simovski, S. Tretyakov, Y. Hao, and C. Parini, "Transmission of images with subwavelength resolution to distances of several wavelengths in the microwave range," *Phys. Rev. B* **77**, 193108 (2008).

Biographies and photographs of the authors not available.